Number Systems With Examples

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1 Decimal To Binary

Conversion of a decimal number with base 10 to a number with base 2 is very simple.

To convert a number from system A to system B we need to divide that number continuosly with the base of system B until further dividing the dividend will cause the remainders to repeat.

Once you have all the remainders write them in a sequence starting from the last remainder (bottom to top)

Let's take few examples

- $\mathbf{(20)_{10}}$

Base divider	Dividend	Remainder
2	20	0
2	10	0
2	5	1
2	2	0
2	1	1
	0	

We will write the final result as $\mathbf{(20)_{10}}$ – $\mathbf{(10100)_2}$

Notice here we stop when dividend is 0 because further dividing it will give us 0s in remainder.

- $\mathbf{(345)_{10}}$

Base divider	Dividend	Remainder
2	345	1
2	172	0
2	86	0
2	43	1
2	21	1
2	10	0
2	5	1
2	2	0
2	1	1
	0	

We will write the final result as $\mathbf{(345)_{10}}$ – $\mathbf{(101011001)_2}$

Hope you are getting a hang of it.

Note The logic remains same for converting a number from decimal to Binary, Octal, Hexadecimal.

Here are some excersises to flex your muscles. Solutions are shared at the end of the pdf

1.1 Exercise Convert Decimal to Binary

1. $\mathbf{(4)_{10}}$
2. $\mathbf{(2)_{10}}$
3. $\mathbf{(35)_{10}}$
4. $\mathbf{(321)_{10}}$

Although examples are not shown give the below challenges a try and see whether or not we can apply the same principle used while converting to other systems

$(24)_{10}$ Convert Decimal To Octal
$(24)_{10}$ Convert Decimal To Hexadecimal

2 Decimal To Octal

Using the same logic as above lets directly jump by trying some examples

- $\mathbf{(30)_{10}}$

Base divider	Dividend	Remainder
8	30	6
8	3	3
	0	

Final result as $\mathbf{(30)_{10}}$ – $\mathbf{(36)_8}$

- $\mathbf{(647)_{10}}$

Base divider	Dividend	Remainder
8	647	7
8	80	0
8	10	2
8	1	1
	0	

Final result as $\mathbf{(30)_{647}}$ – $\mathbf{(1207)_8}$

2.1 Exercise Convert Decimal to Octal

1. $\mathbf{(3)_{10}}$
2. $\mathbf{(8)_{10}}$
3. $\mathbf{(99)_{10}}$
4. $\mathbf{(321)_{10}}$

3 Decimal To Hexadecimal

We will use the same idea used above. Here the base is 16.

– $\mathbf{(789)_{10}}$

Base divider	Dividend	Remainder
16	789	5
16	49	1
16	3	3
	0	

Result will be written as $\mathbf{(789)_{10}}$ – $\mathbf{(315)_{16}}$

3.1 Exercise Convert Decimal to Hexadecimal

1. $\mathbf{(30)_{10}}$
2. $\mathbf{(18)_{10}}$
3. $\mathbf{(99)_{10}}$
4. $\mathbf{(231)_{10}}$

4 Converting Base-2/8/16 to Base-10

We have converted from Binary, Octal and Hexadecimal to Decimal lets see how we can do the reverse.
To obtain the Binary/Octal/Hexadecimal representation we divided the number by the respective base.
To obtain decimal we multiply.
To convert binary to Decimal we follow this procedure.

1. Take a digit going from right to left.
2. Multiply the digit with $\mathbf{2^{n-1}}$ (for the rightmost digit n=1 and n increases as we move towards the left by 1 step).
3. Repeat the step 2 for every digit and take sum of them.

Let's take an example.
$\mathbf{(25)_{10}}$ The representations in other systems are as follows $\mathbf{(11001)_2}$, $\mathbf{(31)_8}$, $\mathbf{(19)_{16}}$
Binary to Decimal conversion will look like -
$2^4 * 1 + 2^3 * 1 + 2^2 * 0 + 2^1 * 0 + 2^0 * 1$
Octal to Decimal conversion will look like -
$8^1 * 3 + 8^0 * 1$
Hexadecimal to Decimal conversion will look like -
$16^1 * 1 + 16^0 * 9$

4.1 Exercise

Convert Binary/Octal/Hexadecimal to Decimal

1. $\mathbf{1001000_2}$
2. $\mathbf{110_8}$
3. $\mathbf{51_{16}}$

5 Binary-Octal-Hexadecimal Relation

Let's take a common example of converting a decimal number to each of the other systems $\mathbf{(100)_{10}}$

Binary

Base divider	Dividend	Remainder
2	100	0
2	50	0
2	25	1
2	12	0
2	6	0
2	3	1
2	1	1
	0	

Result: $(1100100)_{2}$

Octal

Base divider	Dividend	Remainder
8	100	4
8	12	4
8	1	1
	0	

Result: $(144)_{8}$

Hexadecimal

Base divider	Dividend	Remainder
16	100	4
16	6	6
	0	

Result: $(64)_{16}$

Let's see the relation between Binary, Octal, Hexadecimal

Binary has a base of 2 Octal has a base of 8 Hexadecimal has a base of 16

We can write 2, 8 and 16 as $\mathbf{2^1}$, $\mathbf{2^3}$ and $\mathbf{2^4}$ respectively.

This gives us a way to easily convert from either Octal, Hexadecimal or Binary to the other 2 systems.

The exponents of 2 for 2,8,16 gives us the number of bits that are required to represent all of the symbols in that particular system.

So for $\mathbf{2^1}$ with 1 as power to represent all the symbols (0,1) we need only 1 bit.

0, 1 represents all the symbols using only 1 bit

For Octal we need 3 bits, it will look something like

Bit representation	Symbol
000	0
001	1
010	2
011	3
100	4
101	5
110	6
111	7

Let verify by using the results obtained from the above example

So $\mathbf{(100)_{10}}$ gave us $\mathbf{(1100100)_{2}}$ $\mathbf{(144)_{8}}$ $\mathbf{(64)_{16}}$

Let's group 3 bits of the binary representation starting from the extreme right moving towards left and see what does these groups represent individually in binary.

Remember similar to the decimal where 0000600 is same as 600 in other systems also adding 0s at the extreme left does not change the actual value.

$\underline{001}$	$\underline{100}$	$\underline{100}$
1	4	4

By grouping 3 bits of a binary representation we get the Octal representation of the same number.

Similarly for Hexadecimal base **16 or 2^4** we make a group of 4 bits.

0110	0100
6	4

Similarly this can be reversed given a Hexadecimal/Octal representation to get the binary representation we simply take every digit starting from the extreme right and write its individual binary representation.

So given $\mathbf{64_{16}}$ find base-2 representation.

We do

6	4
0110	0100

Giving us 01100100 – 1100100 (trimming extra 0s on left)

So now lets say we design a new system with base 32 then it will contain 32 symbols. 32 can be written as $\mathbf{2^5}$.

Thus to convert a Binary/Octal/Hexadecimal to our new system would be a lot easier now. We only need to group 5 bits from right to left and write its equivalent symbol in the new system

so $\mathbf{64_{16}}$ in base 32 can be written as

1. Convert to binary 1100100
2. Group 5 bits (rtl) 00011 00100
3. Write equivalent symbol 00011 – 3, 00100 – 4

Giving us $\mathbf{(34)_{32}}$

Note This way of converting from Binary< − >Octal< − >Hexadecimal is possible because they are all having same base of 2 and only the power varies.

6 Exercise Solutions

Decimal To Binary

1. 100
2. 10
3. 100011
4. 101000001

Decimal To Octal

1. 3
2. 8
3. 143

4. 501

Decimal To Hexadecimal

1. 1E
2. 12
3. 63
4. E7

Base-2/6/16 To Decimal

1. 72
2. 72
3. 81